From free will to probabilities: the scientific domestication of freedom.

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Abstract

In this paper I will discuss what I will call, analogously to Jan Hacking Taming of chance *(Hacking 1990), the taming or domestication of free will.*
During the period 1850-1890, through the scientific work of several authors, working on such different fields such as physics, economics or statistic, all concerned with the problem of free will, came a progressive identification of probabilities with free will. Quetelet's answer to the problem suggested this analogy for the first time, but also raised a problem: probabilities are the absence of causes, while a free action is up to the one who takes it, and is therefore caused by him. The taming of free will and its identification with hazard will thus regularly appear in the following years, and the two notions will entangle even deeper.

When dealing with James Clerk Maxwell's impressive work on statistical mechanics, one my encounter a curious equation-like statement: atom = individual.
The statement may seem one of the numerous analogies that Maxwell often suggests (Turner 1955; Kargon 1969) but it is probably more than that. As a matter of fact it is a relic of a previous argument. Maxwell inherits of this equivalence through Quetelet's use of statistical reasoning and applies it to his physical reasoning, but also to his philosophical speculations on the matter of free will.
Maxwell was not isolated in transplanting Quetelet social statistics, and particularly Quetelet's understanding of free will outside its main application domain.

When dealing with Walras's economics, one may find a similar concern both in Walras's own writings (Une branche nouvelle de la mathématique Walras 1987, 7:327–328), and in some articles of the time related to the walrasian contribution: some scholars considered Quetelet's "homme moyen" as a plausible interpretation of Walras general equilibrium.

Finally, Joseph Boussinesq infamous contribution to the determinism versus free will debate, while discussing a particular kind of differential equations, also referred to Quetelet.
What did Maxwell, Walras and Boussinesq share? Did they mutually influence each other and how? What's the hidden link that goes from the Belgian statistician Quetelet to an economist, a physicist, and a mathematician?

In this paper, I will defend a bold claim: that Quetelet, an astronomer and a social statistician, attempted to tame the dilemma of determinism by transforming free will into chances. The taming had a prize: free will may be opposed to determinism in some sense, but supposes intentionality. A free decision is a decision that is up to someone. Probabilities are not intentional: they are up to nothing and no one. Therefore, transforming free will into probabilities gives up what free will is really about.

Quetelet's taming of free will had an important influence on the debates that will follow, especially those concerned with the application of mathematical methods.
The process of taming free will is to some extent the parallel of a more famous and well studied historical phenomena, commonly referred as the taming of chance, or the probabilistic revolution (Hacking 1990; Porter 1986; Stigler 1986; Gigerenzer et al. 1989). Nevertheless, free will and probabilities may empirically show similar results, but are totally different from a metaphysical point of view. When dealing with the compatibility between free will and a deterministic science, scientists had to confront with this metaphysical tension.

Moreover, the debate concerning determinism probabilities and free will, entangled with questions concerning rational mechanics, often confusing or mixing together both subjects. In this paper I will focus on the lesser-known, French part of this debate, but to show his entanglement with the more general question of the taming of chance and the taming of free will, I will stress the numerous connexions with the well-studied Victorian debate (Krüger 1987).
The transformation of free will into probabilities is a scientifization of a philosophical question, and involved contributions from several scholars in different disciplines, natural scientists, social scientists and philosophers.

Physics and the social sciences, especially economics, thus shared a common question and contributed to the imprisoning of free will into a probabilistic box.
But as for the proverbial Aladdin's lamp, when scientists polish the box, the jinn escapes and it is each time harder to tame it again.

Laplace's kind of determinism

Let us imagine a powerful entity that at a given instant would know every force, every speed, and every position of any single atom in the universe. And let's imagine this entity to be incredibly skilful in calculation. Then, it would be possible, following the laws of mechanics, to calculate the entire past and future of the universe and express it as a single mathematical formulae, or at least, this is what Laplace expressed in a nowadays-famous statement.
The entity, what will be later called the Laplacian demon, implies three separate assertions about nature, mainly

- That every lump of matter possesses a position and a speed at every instant
- That matter in motion is all that exists
- That one may write The Equation, i.e. a mathematical formula that comprises the movement of those lumps of matter, and that The Equation has one and only one solution.

A modern reader may be tempted to discard such a claim as the megalomania of an old-fashioned scientist. After all, Laplace did not even know how to describe the electro-magnetic forces, not to speak of the structure of atoms, or of the theory of relativity. How possibly could he assume that the universe obeys to such an Equation?
And as a matter of fact, he did not know how to solve most of the differential equations that he encountered in his daily work. Or how to measure with the required precision all those forces, speeds and positions that his entity would require for his calculation. Nor we do, I suppose.
This was not, of course, the point, and Laplace was not such a megalomaniac. His *petitio principii* comes in a treatise on probability (Laplace 1819), i.e. in a book whose aim is to do "the best we could" knowing that we are not such an entity, and we do not measure nor calculate with infinite precision. Laplacian probabilities were the answer

to the epistemological limitation of human beings. Probabilities are the expression of our ignorance and not of the nature of things.

Laplace was therefore suggesting, with his demon, that ontologically matter in motion obeys to strict laws, and that determinism is true. And to some extent he was encouraging a reductionist thinking: if matter in motion is all that exists, then philosophical questions concerning how the world really is must be set at the most fundamental level, that of the motion of atoms.

Determinism was a kind of common hypothesis that was largely shared by the scientific community of the XIX century (Israel 1991). One may find similar deterministic allegiance outside physics (Bernard 2003 (1865), p. 69). It was a common ingredient of scientific enquires to believe that determinism (not necessarily in the Laplacian form) was a kind of scientific requirement, to the point that even Maxwell, while defending free will, recognized that.

The Laplacian determinism will remain a common standard of scientific enquire, and a common reference for those concerned with determinism and free will. But as we will see, it found a new declination when applied to the social sciences.

Quetelet and the problem of free will

Regularity is the most striking property of the motion of planets and stars. It is no surprise that the astronomer Adolphe Quetelet was deeply persuaded of the importance of looking for such regularities in natural phenomena.

Laplace directly influenced Quetelet (Stigler 1986, 162) notably on the deterministic image of the world.

In 1834 Quetelet wrote a letter to Sylvain van der Weyer, concerning a new kind of regularity: the regularity of averages.

> *We will be able to solve the great problems of population like those of celestial bodies; and the more astonishing is the remarkable analogy that exists between the formulae that will be used in computations. I believe that I have almost succeeded in what I am talking about since a long time: the possibility to do a social mechanics like we do a celestial mechanics; to formulate the movements of the social body as*

we do for celestial objects and to identify their properties and laws of conservation (Quetelet to Sylvain van der Weyer, in Armatte 2010, p. 7, August 22 1834).

What Quetelet was trying to do was to interpret the regularity of averages, analogously to physical and astronomical regularities.

But this was not an easy task, and Quetelet was quite aware of a main difference between physical averages and social ones.

One may measure the position of a given celestial object many times and compute the average value of the position of the object. If all the measures take place under similar conditions, one may conclude that the average value is to some extent the *real* position of the star. The difference between the measurement and the average is error. The errors that one inevitably does while measuring, should compensate each other to some extent, and follow a (Gaussian) distribution. Those errors are random, they follow a statistical distribution, but each error has a probability to occur. This is the meaning of the Laplacian probabilities.

Nevertheless, if one considers the average of a group of individuals, probabilities mean something else. Every individual is different; every individual is an object on his own. Contrary to the astronomical measurement, social statistics never measures the same individual twice.

What is then the average result supposed to represent? Not a *real* property of a given man, but, in Quetelet's mind, something as close as possible to that. Quetelet will call the fictional result of this statistical process 'homme moyen' (hereafter *average man*). The *average man* is an abstract and ideal individual; no one corresponds to this average man. In Quetelet's mind it represents the ideal man, both in a positive and normative sense (Stigler 1986, 169–174).

Another important point is at stake: Quetelet measures the average behaviour of individuals and tries to identify the 'laws that regulate the facts'. But the regularity of averages strongly suggests that some kind of external forces act on the general behaviour, something that seems to be in contradiction with the freedom of the will (Desrosières 1993, 99–104).

Quetelet will explain his own views on the question of the compatibility between statistical regularities and free will (Quetelet 1847).

He clarifies that 'the possibility for moral statistic to establish and deduce useful consequences, depends on this fundamental fact, that free will vanishes and has no sensible effect if the observations concern a large number of men'.

This is a bold taming of free will: freedom of the will, when one considers large groups of individuals, becomes a probability, and follows a statistical distribution, the Gaussian function. That's why it can vanish: because it is equally distributed on both sides of an average value. Therefore, from the point of view of statistical treatment, free behaviours are random behaviours. Free will is nothing more than a probability.

Those probabilities are not anymore the probabilities of Laplace. As Desrosières (1993, 100) says Quetelet has a frequentist interpretation of probabilities, and suggests that those probabilities expresses genuine random events, rather than ignorance of true causes.

This is to some extent a denying of reductionism: it is not possible for the statistician, to talk about the individual. Only large group of individual obey to statistical laws. Individual acting freely know what they do and why, they have intentionality; but for the statistician it does not matter which individual does a given choice. It only matter that in average they are predictable. Quetelet says that, 'the action of man is reduced into such a domain that the great laws of nature are not under his influence'. But the converse is also true: the laws of nature cannot predict the individual behaviour. Quetelet believes that humans obey both to individual and to social compulsions. Individual compulsions are randomly distributed around a mean and follow a Gaussian distribution; the value of the average and the shape of the Gaussian are constrained by social habits.

Quetelet was quite careful on the question of free will, and never denied his reality. Others scholar were much less cautious.

Especially, Henri Thomas Buckle, in his *History of civilization in England* (hereafter HCE) stated that history followed deterministic laws, that those laws allowed a scientific understanding of history, and that "the preceding proofs of our actions being regulated by law, has been derived from statistics" (Buckle 1884 (1864), 24). Buckle considered that the science of history was on a path of progression that would ultimately reach the same degree of certainty than physics and astronomy.

Buckle's claims were close to those of Quetelet, but lacked his rhetorical modesty. It is therefore unsurprising that reactions against Buckle were even more vigorous than against Quetelet. Echoes of these reactions may be found in several statistical books and papers, years after the first publication of the first volume of HCE in 1857. Pages and sometimes chapters are devoted to explain to what extent statistical regularities are compatible with free will.

Economics and free will

Economics as well had to confront the dichotomy of determinism and free will, but as Ménard (1987) argues, probabilities did not play a major role in the mathematization of economics. Economics was much more influenced by a mechanistic view of nature inspired by physics: even those economists such as Cournot that had a considerable knowledge of probability theory, did not attempt to use it in the description of economic phenomena.

Mosselmans (2005) argues that Adolphe Quetelet's influence on theoretical economics should be considered as significant, especially on Jevons and on the German historical school.

Jevons as Cournot before and Walras after him did use probability theory as a justification of the continuity of the demand and supply function through the law of large numbers but did not use probability theory in economics.

The law of large numbers plays a double role: on one side it implicitly introduces the theory of probability into mathematical economics, thus allowing a mathematical form inspired by xix century deterministic physics (Mirowski 1989). On the other, it permits to some extent to neglect the problem of free will: using large groups of individuals and Quetelet's hypothesis one may avoid the deterministic dilemma.

But what is at stake is not simply the regularity of averages. It has been noted by Zamagni and Screpanti (2000, 172–173) that the marginalist revolution abandons "collectives subjects", social classes, or political bodies, in favour of individuals.

It is true that the continuity of demand and supplies curves rely on the hypothesis of large groups of individuals, but at the same time, mathematical economics focuses on idealized individual-based models. Contrary to Quetelet, who never discussed the individual behaviour, the marginalists stressed both the individual and the aggregate level, trying to move from one to the other.

Economics was therefore at the core of a tension between the Queteletian argument of mutual compensation of individual wills and the more classical deterministic picture inherited by Laplace's mechanics.

We will now discuss one of the most important contributions of the time to mathematical economics, namely Léon Walras general equilibrium and his involvement in this debate. Walras cites Quetelet's work in his *Une branche nouvelle de la mathématique: De l'application des mathématiques à l'économie politique.* He refers to the regularity of averages in the context of the 'freedom of the will, that can not be submitted to mathematical calculus (ibid. p. 325)'. Walras is referring to his presentation of his *Principes d'une théorie mathématique de l'échange* in Paris, during a session of the *Académie des sciences morales et politiques* (Vergé 1874).

The presentation was a complete failure for Walras: as he relates himself, Levasseur attacked him on the basis that "freedom can't be put into an equation".

The proceeding of the encounter tells a more complex story. Levasseur was afraid of "the danger represented by taking as a unity those things who are complex in their essence, and in applying to political economy an excellent method for the physical sciences, but a method who could hardly be applied without care on those phenomena whose causes are variable, complex and especially subjected to this variable cause, irreducible to algebraic formulae: human freedom".

Walras's answer to Levasseur in his *De l'application des mathématiques* suggests that statistics implies the possibility to express the laws of human behaviour in mathematical form. Regularities allow discovering those laws. Walras is therefore suggesting that his method is as dangerous to human freedom as the one cherished by Levasseur, which sounds as a cogent argument.

Walras own attitude concerning the individual and the aggregate level of description is a tricky and controversial subject: Walras seems to endorse a singular synthesis of individualism and holism (Lallement 2014). Nevertheless, Levasseur may have understood Walras in a very individualistic sense, and as such as in contradiction with the non-reductionist attitude of statisticians on the matter of free will. Levasseur as a statistician was probably directly concerned by the debates on the compatibility between statistical regularities and free will, and reacted to Walras supposed reductionism.

Human freedom, a paradigmatic example of a varying cause, is in his views irreducible to algebraic formulae that directly involve individuals. Statistics may works, due to his capability of taming free will as probabilities, but only when groups of individuals are concerned.

Of a particular interest is the reaction of Flechey (1874) in the *Journal des économistes.* The French author compares two papers, one of Quetelet (Quetelet 1873), another by Engel (Engel 1871). Flechey discusses the case of the effects of individual actions at a social level and stresses particularly that "the laws of conservation [of the moral realm …] may be considered as the effects produced by random causes (p. 37)" thus suggesting an equivalence of free and random behaviours, of free will and probabilities, under the constraint of a large number of individuals. He considers that "free will participates in a random way to the collective action (p. 38)". Flechey is showing that the antireductionist hypothesis of Quetelet, i.e. that the individual is free but the collective behaviour is not, requires to consider that individual actions balance each other in the global regularity. As a consequence, the intentionality (the will) in the action of an individual is completely erased by the statistical treatment. It is therefore impossible to deduce the individual action from the collective behaviour. The laws of society act at a different level than the individual decision and individual acts do not affect social regularities.

Flechey also considers Engel's contribution as showing to what extent the individual action is influenced and shaped by social constraints. Nevertheless, says Flechey, when we consider several free actions taken together, one has to compare them, "to give to every element a convenient coefficient", i.e. a numerical comparison between individuals. Such a comparison, Flechey concludes, cannot be done.

Interestingly Flechey cites the "recent answer by Wolowski and Levasseur to Walras" (p. 42-43) against the use of mathematics in economics. Walras's mathematics, says Flechey, describes the individual willingness to buy and sell, and wants to deduce from the individual propensity the general law of demand and supply. But, continuous Flechey, it is not possible to mathematically express the moral attitude of individuals. Therefore Walras attempt to express "a scientific formula of demand and supply" based on the "individual propensity to buy and sell" fails.

A gentler reader of Walras, Fauveau (1882), tries to interpret economic agents not as real individuals but as 'average men' (*hommes moyens*). Fauveau wants to defend the use of mathematics in economics, and refers to Jevons, Walras, Cournot and others, again linking Walras and Quetelet through the average man and the problem of free will.

Fauveau affirms that it is possible to formulate general laws of human behaviour, despite the existence of free will. The laws of human behaviour "must rely on the common characters of every man; therefore it is necessary to study this ideal being that we call *homme moyen*."

To do such a hypothesis is nonetheless equivalent to accept a 'first approximation' of real individuals: to some extent it implies that the average man expresses the core of human behaviour and ignores 'frictional aspects' analogously to the physicist attitude, when one ignores friction and perturbations in his models. It also implies that free will is a negligible aspect of human behaviour that can be ignored as a first approximation.

Fauveau is fully consequent on this point since "for the common objects of consumption, *the freedom of the will is strongly limited [...]* We thus understand how it is possible that the acts of the *average man* can obey to *constant laws, sometimes mathematical ones* (ibid, p. 263). Fauveau is therefore advocating a mechanical understanding of human beings.

Levasseur too expressed himself on the topic of mathematics and free will in other occasions than against Walras, always showing scepticism toward the use of mathematics applied to economics (see e.g. (Levasseur 1898, 295 – 296), for an historical discussion of Levasseur's views on mathematics see (Mueller 2015a)).

He insists for instance on the fact that humans are both observers and observed in the social sciences: therefore “a reaction happens, as a consequence of the observation of phenomena, over the development of those phenomena, [...] and we can affirm that, for the moral sciences, humans do not solely study the laws of their own nature, but learn to better conform their private conduct and the public institutions to those laws (Levasseur 1869, 124)”.

Levasseur is thus suggesting that the laws of society change the social phenomena under study, something that the mathematical instrument is unable to do.

More simple criticisms, comprehend arguments about the incompatibility of the method of physics, mathematics, and the complexity of society. Levasseur particularly noticed that it is impossible in the social sciences to isolate a given phenomena, as it is often the case in the physical sciences.

Léon Walras, the main victim of this reaction against mathematics, never denied the role of free will in the social sciences: he even considers free will as constitutive of the human nature (Walras 1988, 39).

Walras believes that human actions are free: the demand and supply are simply not up to us. They are consequences of the natural state of thing, and we must conform to them, despite our will. He gives a parallel with gravitation: “The fact that gravity is a natural fact, obeying to natural laws, does not imply that we have to let it act passively, we can resist to it or let it act depending on our convenience; but we can not change its nature or its laws. […] The same is true for the [exchange] value” (Walras 1988, p.50). Natural facts escape the control of human will, and are, analogously to gravitation, a suitable topic for mathematization. But this does not imply that one is denying free will.

That Walras’s mathematical approach to economics suggests a deterministic image may have been encouraged by the numerous analogies that Walras himself shows between economics and physics. Walras draws parallels between economics and astronomy (Walras 1987, 7:291–329), he compares speed, time and space with *rareté* (scarcity), quantity and utility (Jaffé 1965, p. 398, letter 275 to Jevons, 23.5.1874)

Others scholars than Levasseur criticized the mathematical approach of Léon Walras (see for instance (Ridolfi 1996; Van der Rest 1996)), and expressed mixed feelings on his use.

The introduction of algebra and calculus in the discourse, thus imitating physics, implies a certain (at least perceived) reductionism from the aggregate society to the individual behaviour. It may invalidate Quetelet's solution to the problem of free will. Walras is therefore rejected as denying free will, or, when accepted, interpreted trough the metaphor of the average man, in order to escape the free will versus determinism dilemma. Taming free will in the form of probabilities requires large numbers, but also demands to focus on the average behaviour, forgetting the individual agent. When individuals are important, the problem of free will surfaces again.

Heat, demons, and statistical knowledge

Henri Thomas Buckle's claims about determinism and social laws did not go unnoticed in the physicist's community. For instance, James Clerk Maxwell reports "One night I read 160 pages of Buckle's History of Civilisation – a bumptious book, strong positivism, emancipation from exploded notions, and that style of thing, but a great deal of actually original matter, the true result of fertile study, and not mere brain-spinning" (letter to Campbell, Maxwell 1990, 1:576–577, 22 December 1857).
Maxwell was a faithful Christian (Campbell and Garnett 1882), and did not appreciate arguments against free will: it would have been hard to reconcile the notion of merit and grace with a deterministic view of the physical world. But Maxwell was also personally facing a difficult situation: on one side his work on statistical mechanics was a paradigmatic example of the incredible efficacy of a mechanistic view of nature, somehow confirming the Laplacian dream. On the other, agnostic scientists, such as Tyndall or Huxley, were using scientific results to discredit faith and religion in favour of a scientific understanding of nature (Porter 1986; Porter 1981). Maxwell felt uncomfortable with some of the "positivistic" views, as he called them, expressed by Buckle, Huxley or Tyndall.

His direct concern with Quetelet's dilemma comes from his introduction of statistical reasoning into the dynamical theory of gases (Maxwell 1860). Maxwell was inspired by a review written by John Herschel of Quetelet's *Letters sur la théorie des probabilities* (Herschel 1850). In his book review Herschel gave an example of a ball falling from a given high on the ground. If we repeat the experience several times and record the different results, says Herschel, we may find a Gaussian distribution. Such an example is a physical one, but the understanding of the statistical dispersion is of the Queteletian kind: every record is a different event, and the average results do not correspond to any 'real' event'.

As a matter of fact, the introduction of statistics in his kinetic theory of gases implies a double standard of reasoning, analogous to the economist dilemma: on one side the equation of thermodynamics, the laws of gazes or the second principle, follow a strict determinism. On the other, the deep nature of things is statistical: it is the law of large numbers that allows a continuous deterministic description of macroscopic regularities. But Maxwell is able to do something more: atoms are invisible theoretical entities, whose dynamical laws are unknown. The statistical treatment only requires atoms to be subject to a probabilistic description, regardless of the true nature of those probabilities. They could either be subjective probabilities, thus suggesting that the true dynamic of atoms is deterministic, or objective probabilities, thus suggesting that atoms really obey to indeterministic laws. Maxwell is therefore suggesting an agnostic view, signifying that the scientific understanding of nature cannot conclude on the matter of determinism. He will prove to be fully conscious of this ambiguity while discussing the controversial issue of free will.

In 1873 Maxwell delivered his own view on the compatibility between science and free will in front of the Eranus Club.

Maxwell tried to show that the knowledge of nature that we have is analogous to the Queteletian knowledge of averages and laws of society.

Maxwell distinguished between 'dynamical knowledge' and 'statistical knowledge'. Dynamical knowledge follows individuals in details, while statistical knowledge considers averages over groups. What Maxwell pretends is that our knowledge of physical bodies may be only of the statistical kind.

As such, physical laws are themselves of a statistical kind: they may be compatible with a deterministic behaviour of atoms, but analogously to the human behaviour they are also compatible with the utmost irregularity at the atomic level.

As a general rule, the irregular behaviour of the atomic level should be of no influence on the macroscopic description, but Maxwell knew very well of the existence of amplification phenomena able to transport some of those irregularities from micro to macro dimension.
Analogously to atoms moving randomly – says Maxwell - human behaviour can, some now and then, have an impact even at the level of societies in a complete unpredictable way. This unpredictability is the consequence of a new idea that was going to be later called sensibility to initial conditions.

> *When the state of things is such that an infinitely small variation of the present state will alter only by an infinitely small quantity the state of some future time, the condition of the system, whether at rest or in motion, is said to be stable; but when an infinitely small variation in the present state may bring about a finite difference in the state of the system in a finite time, the condition of the system is said to be unstable. (Maxwell 1995a, 2:819)*

Indeterministic "swerves" may be amplified by the sensibility to initial conditions like a "pointsman on railway with perfectly acting switches who should send the express along one line and the goods along another (Letter to John William Strutt Maxwell 1995a, 2:583 december 1870)".
The recurrent pointsman metaphor in Maxwell writings (Stanley 2008) symbolizes the possibility to transport infinitesimal violations of determinism from a microscopic to a macroscopic dimension.

Something is nonetheless missing if one wants to give free will a real chance to exist. The Maxwellian picture of molecular motion, a form of agnosticism, still relies on probability theory.
Probabilities may provide swerves, but still random swerves, not intentional ones. What is necessary to defend free will, is to provide a swerve that is up to us.

Maxwell may have not been fully satisfied with his answer to determinism even from a physicist point of view. He was very conscious that he lacked a source of indeterminism that was compatible with the laws of mechanics. One thing is to say that we do not have a direct access to the observation of molecular motion, and another is to provide an example of a possible motion that is not deterministic. Maxwell was not without knowing that every physical law of his time was in fact deterministic.
What was missing was the source of a swerve, or to use Maxwell's metaphor, what was missing was the mechanical equivalent of the pointsman's action: a small perturbation that may count as a probability from a macroscopic point of view, but that is up to the agent will.
His death in 1879 prevents us from knowing the following development of his own ideas on the topic, but in a letter to Galton (Maxwell 2009, 3:756–758, 26.2.1879) he expressed very positive feelings toward a quite astonishing idea by Joseph Boussinesq, a French mathematician.
What Boussinesq had found was in fact what Maxwell was in search of: a way to 'pick a hole' in the determinism of Newtonian mechanics.

Singular solutions: a way to pick a hole in the determinism of mechanical laws

What Boussinesq had figured out, and Maxwell found so interesting was a strange and unexpected property of a class of differential equations: under some sets of initial conditions there are certain differential equations that have more than one solution.

The modern reader will not be much surprised: a differential equation must obey to certain properties of smoothness, specified by the Lipschitz continuity in order to have a unique solution. But Boussinesq was much surprised by the result: he was astonished to discover that differential equations may have more than one solution, and especially differential equations that were equations of motion of conceivable physical systems.

Boussinesq published a quite long *mémoire* (1879) that was the improved version of previous communications (Boussinesq 1877a; Boussinesq 1877b; Boussinesq 1877c; Boussinesq 1878). His main points were that

- There are physical problems whose equations of motion have more than one solution.
- Those systems require the introduction of a non-physical principle called the director power, which can choose between potential paths to be followed.
- The director power is entirely consistent with physical laws.
- The director power may act only at very special points, called singular solutions, at which the system is undetermined: those points have a higher chance to exist in more complex system, especially in living beings.

One may notice that 1) is simply the converse of the third Laplacian requirement: The Equation has more than one solution.
The director power answers to the double requirement of being indeterministic from a physical point of view, and of being intentional. It therefore matches with the necessities of being up to us and at the same time of respecting Quetelet's requirement of mutual compensation when several individuals are considered.

Boussinesq feels concerned with Quetelet's formulation of the problem of free will. In a paragraph of his *mémoire* Boussinesq clearly states that

> *One of the strongest objections against the doctrine of moral freedom has been the one that one may deduce from the constancy, or at least the slow variation in the number of crimes, individual acts of different kind that a large number of individuals produces every year. […] Between the causes that participate in their formation we may notice only those who always act in the same direction. Moral freedom is not of this kind, […] It is therefore natural that its effect grossly vanishes due to the large numbers collected by statistics.*
> *(Boussinesq, 1879a, p. 61-62)*

Boussinesq is thus conscious of the Queteletian problem, but considers that it can be overcome by denying reductionism. The stability of averages is a property of large numbers, and the effects of moral freedom compensate each other when large numbers are taken into account.

But at the same time, in order to defend the freedom of the will, the denying of reductionism goes side by side with the necessity of a “swerve”, a source of indeterminism at the microscopic level.
What Boussinesq wants to show is an explicit example of an exact knowledge - we may call it ontological - that infirms the mechanical, Laplacian determinism. He’s looking for an exception to such a determinism that may justify his belief in free will.

There is an important technical aspect that links Maxwell with Boussinesq and that is close to the sensibility to initial conditions problem: Boussinesq singular solutions exist only on unstable equilibria, i.e. at very special points, and are close to Maxwell’s idea of a pointsman. A very small cause, in Boussinesq theory a zero cause (the director power require no physical cause at all) suffices to generate a considerable effect. At the same time, the laws of mechanics are not violated and continue to rule the behaviour of almost every object at almost every time. Boussinesq theory seems therefore to be the right answer at the right place.

In order to deduce from the existence of multiple solutions the existence of multiple possible futures, Boussinesq must accept an ontological commitment to mathematical entities. He does, and he’s conscious of this choice: “"[T] he laws of physics and chemistry are reducible [...] to differential equations (Boussinesq 1879, 35)”

It will be the famous mathematician Joseph Bertand himself that will sharply attack Boussinesq on this point (Bertrand 1878). Bertrand will especially focus on some example of mathematical systems with more than one solution that correspond to highly idealized physical systems (such as a perfect symmetric table with more than three legs on a perfect smooth surface, whose reaction force on every leg is undetermined). Bertrand’s interpretation is that highly idealized systems do not exist physically, and that one may understand mathematical equations *cum grano salis*.

Boussinesq also considers physical laws as expressing how the real world is, i.e. ontologically. This point is questioned in a series of papers by Charles Renouvier (Renouvier 1882a; Renouvier 1882b; Renouvier 1882c).

Renouvier's argument is the reverse of the medal, in some sense, of Bertrand's one. Instead of attacking the seriousness of singular solutions and the role of mathematics with respect to physics, he attacks the discourse premise, namely that physical laws has to be interpreted ontologically. This is a necessary argument for Boussinesq: it is because he thinks that physical laws are true-statements about the real world that he has to conciliate the determinism of this laws with free will. But, says Renouvier, one may simply consider that scientific laws are descriptions, and deny to them an ontological commitment. That will suffice to solve the tension between determinism and free will.

Such an argument may sound quite positivistic in nature, but it seems that Renouvier inspiration was more of a Kantian nature: free will is noumenal, while scientific laws are phenomenal, and the two realms simply don't match.

Finally, Boussinesq tries to show that singular solutions have a chance to influence the dynamic of physical systems, and especially of living beings.

He will be criticized on that point by Ignace Carbonnelle, whose objections is simple but striking (Carbonnelle 1879a; Carbonnelle 1879b). Singular solutions happen only on instable equilibria, and instable equilibria have a zero chance to happen: any small perturbation will destroy them, and small perturbation seems to be always available.

It may happen, says Carbonnelle that our brain being on a singular solution, the rest of the universe influence on it compensates and therefore vanishes. But other brains exist that may also be on singular solutions, in order to take a free decision. Would it be the case, they may jump out of this singular state at any instant and perturb our brain. We will then take a decision that will be up to them and not up to us. To deny this possibility is to deny the free will of other agents. Therefore singular solutions are not a good basis for free will.

We therefore see that Boussinesq singular solutions are not without raising questions and doubts: Boussinesq merit is to provide an impressive example of indetermination in the equations of motion, but the implication of an indeterminism in nature is far from being a trivial consequence. Moreover, Boussinesq attempt disclose the dubious premise of the discourse, that mechanicism being a deterministic theory, one may

conclude that determinism is true of the entire universe (modulo some particular exceptions).
Boussinesq's singular solutions were followed by a quite conspicuous amount of papers dealing with the sensitive subject of free will and mechanical determinism (Tannery 1879a; Tannery 1879b; Grocler 1882; Naville 1879; Delboeuf 1882a; Delboeuf 1882a; Delboeuf 1882b), sometimes entangled with debates concerning statistics and free will.
As a matter of fact, the two discussions were not clearly distinguished, and the reductionist dilemma, that characterized to some extent both approaches, may have increased the confusion. Nevertheless, the focus of the discussion quickly moves on a different debate, concerning the conservation of energy (the first principle of thermodynamic see (Mueller 2015b) for a complete discussion).

Boussinesq attempt to solve the dilemma of free will offers the entire set of required arguments: it is compatible with physics and with science in general, it is up to us and allows intentionality, but fails to be a credible solution due to his being a too small of a swerve, in some sense. Boussinesq's free will has a zero chance to happen, and zero is too law a chance to tame free will.

Renouvier, James and the taming of free will

One of the finer critics of Boussinesq's paper that was even published as a complement to Boussinesq 1922 re-edition of his *mémoire* was, as we mentioned, Charles Renouvier. Renouvier was a French philosopher, deeply concerned with the debate on free will from a Kantian perspective.

In an 1880 paper on his journal *La critique philosophique* while discussing Quetelet's solution to the problem of free will, he considered the following example.
Let's draw some black and white balls from an urn, and record the colour of the ball. The long run proportion of white and black balls (say 1 white every 10 blacks) suggests that there is some cause that determines this proportion. Nonetheless the belief that the balls in the urn are in a 1 to 10 proportion before the extraction, or that they

acquire their colour at the moment of the extraction with the appropriate chances are equally credible on the basis of empirical observation.

Renouvier then affirms, “we cannot legitimately conclude that a particular individual was predetermined in his action, analogously to the black ball having is colour before extraction”. Renouvier is therefore advocating for a genuine indeterminism, a completely new understanding of the notion of cause as Krüger (1987) defends. He’s thus suggesting that probabilities may account for free choices, and that freedom of the will is indistinguishable from a random probability in a scientific account of the world.

One should be careful not to over interpret Renouvier: the aim of his example is to show that, in a statistical record of events, the action of a free choice is indistinguishable from a truly random event, and that a genuine random event is indistinguishable from an epistemic probability, Therefore, probability as ignorance, probability as indeterminism and free will are blurred together when it comes to statistical records.

Nonetheless, Renouvier maintains the conceptual difference between a free act, a random act, and ignorance. Even if the three are expressed in the form of probabilities when it comes to statistical accounts, they are nonetheless conceptually different.

A regular contributor of *La critique philosophique* and close Renouvier’s friend was a young William James.

Renouvier exerted an important influence on William James as the American philosopher notes in his own diary (April 30, 1870, see (Perry 1948, 121)) and their collaboration with numerous papers on the Revue philosophique, lasted for almost a decade.

There is an important correspondence between the two men, the younger James always showing great respect for Renouvier’s idea, but never hesitating in stating his own views. Renouvier’s discusses with James on free will in many occasion, and in one he mentions his on-going work on Boussinesq (James and Renouvier 1929, 23).

James was deeply concerned by the problem of free will, and discussed the matter in the *Revue philosophique* in several occasions. Particularly, he addressed questions concerning the importance of free will for moral responsibility and as a central factor for social change. In his paper *Great Men, Great Thoughts, and the Environment* (James

1880), translated by Renouvier (James 1881), he defended a view of history centred on the free action of individuals, against the Spencerian view of environment as the motor of history. He cites Buckle as a further example of environmental determinism, and refuses the Spencerian views as a *non sequitur*. James considers that the environment can maintain and amplify certain changes that happen in the course of history, while it cannot create them. The creation of novelty comes from the individual, free action. This is analogous, James says, to the Darwinian mechanism of natural selection: the environment selects those novelties that fit with it, but the novelties come from individual changes.

Again, what is at stake is an analogy with the free action of individuals: by acting one way or another, individuals may initiate an entirely new path for history: the environment will then select it or not depending on deterministic reasons. But the initial element of novelty is entirely up to his creator.

In his paper *The dilemma of determinism* (James 1884b) also translated by Renouvier (James 1884a), James defends a view of freedom of the will as the potential possibility of different possible worlds, in which free choice act as a selection of possible alternative futures. But, considers James, such a list of alternative possibilities is just another way of naming hazard.

Doyle (2010) considers William James as the first philosopher proposing a two-stage model of free will, in which indeterminism creates possibilities, why determinism select between possible futures.

What is clear is that James considers free will as analogous to hazard, and identifies random choices with free choices: the taming of free will is now complete.

Conclusions

Free will became a matter of interest for those concerned with statistical regularities due to the work of Adolphe Quetelet. Statistical regularities may seem to be in contradiction with free will, and Quetelet had to stress the compatibility between random probabilities and statistical regularities. From that moment on, free will started to be identified with probabilities and hazard.

Nevertheless, such an account entirely miss the intentional aspect of free will. How can we consider that a given choice is up to us, if it is random? While this concern was not directly important for Quetelet, since he was dealing with averages and large groups of individuals, it was a matter of concern for scientists trying to build a bridge between individual action and the collective behaviour. Walras, despite what his own view on the matter may have been, was at the core of a similar dispute. Economists started to use mathematical instruments resembling those of mechanical physics, mainly calculus, but had to interpret them in a statistical and Queteletian sense, both in order to justify the continuity of function and to tame the problem of free will. Thus, they contributed to mix the debate on determinism coming from the natural and the socials sciences. Mirroring this crossing over of arguments, Maxwell showed how even physical knowledge, while using continuous function and claiming absolute knowledge, may in fact be of a statistical nature. Once again, his argument was related to the problem of free will and Maxwell clearly showed the links between his belief in free will and indeterminism.

Maxwell attempt provided a possibility for agnosticism on the matter of determinism, but lacked a genuine source of indeterminism. Boussinesq's *mémoire* provided such a source, and despite his major conceptual problem attracted an important attention. Both authors contributed to the identification of free will and indeterminism, and thus with probabilities, the obvious mathematical instrument to deal with hazard and indeterminism.

Finally, it was Renouvier that stated explicitly that free choices are indistinguishable from random choices; William James, under his direct influence, even considered this indistinguishability as an analogy, and suggested an understanding of free will as a mechanism composed of both an indeterminist and a determinist mechanism, mirroring natural selection.

Such an identification of free will and indeterminism is still common in present philosophical debates (Kane 2002), while the tendency to mix free will and indeterminism in physics is a quite common on-going temptation, despite several criticisms (see Earman (1986 chapter 1)).

The historical route that allowed the taming of free will may explain this unusual blurring of concepts: the taming of chance, as Hacking (Hacking 1990) calls it, developed side by side with a taming of free will, thus creating an historical entanglement of those concepts.

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